

## Knots And Links

*Over the past 20-30 years, knot theory has rekindled its historic ties with biology, chemistry, and physics as a means of creating more sophisticated descriptions of the entanglements and properties of natural phenomena--from strings to organic compounds to DNA. This volume is based on the 2008 AMS Short Course, Applications of Knot Theory. The aim of the Short Course and this volume, while not covering all aspects of applied knot theory, is to provide the reader with a mathematical appetizer, in order to stimulate the mathematical appetite for further study of this exciting field. No prior knowledge of topology, biology, chemistry, or physics is assumed. In particular, the first three chapters of this volume introduce the reader to knot theory (by Colin Adams), topological chirality and molecular symmetry (by Erica Flapan), and DNA topology (by Dorothy Buck). The second half of this volume is focused on three particular applications of knot theory. Louis Kauffman discusses applications of knot theory to physics, Nadrian Seeman discusses how topology is used in DNA nanotechnology, and Jonathan Simon discusses the statistical and energetic properties of knots and their relation to molecular biology.*

*Rolfesen's beautiful book on knots and links can be read by anyone, from beginner to expert, who wants to learn about knot theory. Beginners find an inviting introduction to the elements of topology, emphasizing the tools needed for understanding knots, the fundamental group and van Kampen's theorem, for example, which are then applied to concrete problems, such as computing knot groups. For experts, Rolfesen explains advanced topics, such as the connections between knot theory and surgery and how they are useful to understanding three-manifolds. Besides providing a guide to understanding knot theory, the book offers 'practical' training. After reading it, you will be able to do many things: compute presentations of knot groups, Alexander polynomials, and other invariants; perform surgery on three-manifolds; and visualize knots and their complements. It is characterized by its hands-on approach and emphasis on a visual, geometric understanding. Rolfesen offers invaluable insight and strikes a perfect balance between giving technical details and offering informal explanations. The illustrations are superb, and a wealth of examples are included. Now back in print by the AMS, the book is still a standard reference in knot theory. It is written in a remarkable style that makes it useful for both beginners and researchers. Particularly noteworthy is the table of knots and links at the end. This volume is an excellent introduction to the topic and is suitable as a textbook for a course in knot theory or 3-manifolds. Other key books of interest on this topic available from the AMS are ""The Shoelace Book: A Mathematical Guide to the Best (and Worst) Ways to Lace your Shoes"" and ""The Knot Book"".*

*"Knot theory is a fascinating mathematical subject, with multiple links to theoretical physics. This encyclopedia is filled with valuable information on a rich*

*and fascinating subject." – Ed Witten, Recipient of the Fields Medal "I spent a pleasant afternoon perusing the Encyclopedia of Knot Theory. It's a comprehensive compilation of clear introductions to both classical and very modern developments in the field. It will be a terrific resource for the accomplished researcher, and will also be an excellent way to lure students, both graduate and undergraduate, into the field." – Abigail Thompson, Distinguished Professor of Mathematics at University of California, Davis Knot theory has proven to be a fascinating area of mathematical research, dating back about 150 years. Encyclopedia of Knot Theory provides short, interconnected articles on a variety of active areas in knot theory, and includes beautiful pictures, deep mathematical connections, and critical applications. Many of the articles in this book are accessible to undergraduates who are working on research or taking an advanced undergraduate course in knot theory. More advanced articles will be useful to graduate students working on a related thesis topic, to researchers in another area of topology who are interested in current results in knot theory, and to scientists who study the topology and geometry of biopolymers. Features Provides material that is useful and accessible to undergraduates, postgraduates, and full-time researchers Topics discussed provide an excellent catalyst for students to explore meaningful research and gain confidence and commitment to pursuing advanced degrees Edited and contributed by top researchers in the field of knot theory*

*This book is a survey of current topics in the mathematical theory of knots. For a mathematician, a knot is a closed loop in 3-dimensional space: imagine knotting an extension cord and then closing it up by inserting its plug into its outlet. Knot theory is of central importance in pure and applied mathematics, as it stands at a crossroads of topology, combinatorics, algebra, mathematical physics and biochemistry. \* Survey of mathematical knot theory \* Articles by leading world authorities \* Clear exposition, not over-technical \* Accessible to readers with undergraduate background in mathematics*

*The physical properties of knotted and linked configurations in space have long been of interest to mathematicians. More recently, these properties have become significant to biologists, physicists, and engineers among others. Their depth of importance and breadth of application are now widely appreciated and valuable progress continues to be made each year. This volume presents several contributions from researchers using computers to study problems that would otherwise be intractable. While computations have long been used to analyze problems, formulate conjectures, and search for special structures in knot theory, increased computational power has made them a staple in many facets of the field. The volume also includes contributions concentrating on models researchers use to understand knotting, linking, and entanglement in physical and biological systems. Topics include properties of knot invariants, knot tabulation, studies of hyperbolic structures, knot energies, the exploration of spaces of knots, knotted umbilical cords, studies of knots in DNA and proteins,*

and the structure of tight knots. Together, the chapters explore four major themes: physical knot theory, knot theory in the life sciences, computational knot theory, and geometric knot theory.

The closed orbits of three-dimensional flows form knots and links. This book develops the tools - template theory and symbolic dynamics - needed for studying knotted orbits. This theory is applied to the problems of understanding local and global bifurcations, as well as the embedding data of orbits in Morse-smale, Smale, and integrable Hamiltonian flows. The necessary background theory is sketched; however, some familiarity with low-dimensional topology and differential equations is assumed.

In this book, experts in different fields of mathematics, physics, chemistry and biology present unique forms of knots which satisfy certain preassigned criteria relevant to a given field. They discuss the shapes of knotted magnetic flux lines, the forms of knotted arrangements of bistable chemical systems, the trajectories of knotted solitons, and the shapes of knots which can be tied using the shortest piece of elastic rope with a constant diameter. Contents: *Ideal Knots and Their Relation to the Physics of Real Knots* (A Stasiak et al.) *Knots with Minimal Energies* (Y Diao et al.) *The Writhe of Knots and Links* (E J Janse van Rensburg et al.) *Entropy of a Knot: Simple Arguments About Difficult Problem* (A Yu Grosberg) *Knots and Fluid Dynamics* (H K Moffatt) *Möbius-Invariant Knot Energies* (R B Kusner & J M Sullivan) *Fourier Knots* (L H Kauffman) and other papers  
Readership: Mathematicians, physicists, chemists and biologists.

Keywords: Knots; Topology; Theory of Knots; Energy of Knots; Knot's Invariants; Geometry of Knots; Physics of Knots  
Reviews: "The authors of the articles in this book manage to put together a wide variety of ideas related to the notion of a simple representation of a knot." *Mathematical Reviews*

[Functorial Knot Theory](#)

[The Branched Cyclic Coverings of 2 Bridge Knots and Links](#)

[Including Applications to the Life Sciences](#)

[Linknot](#)

[Knot Theory by Computer](#)

[An Introduction to Arithmetic Topology](#)

[Gauss Diagram Invariants for Knots and Links](#)

[Algebraic Surgery in Codimension 2](#)

[Handbook of Knot Theory](#)

[Hyperbolic Knot Theory](#)

[Knots in Hellas, International Olympic Academy, Greece, July 2016](#)

This textbook contains ideas and problems involving curves, surfaces, and knots, which make up the core of topology. Carlson (mathematics, Rose-Hulman Institute of Technology) introduces some basic ideas and problems concerning manifolds, especially one- and two- dimensional manifolds. A sampling of topics includes classification of compact surfaces, putting more structure on the surfaces, graphs and topology, and knot theory. It is assumed that the reader has

a background in calculus. Annotation copyrighted by Book News Inc., Portland, OR.

This introductory volume provides the basics of surface-knots and related topics, not only for researchers in these areas but also for graduate students and researchers who are not familiar with the field. Knot theory is one of the most active research fields in modern mathematics. Knots and links are closed curves (one-dimensional manifolds) in Euclidean 3-space, and they are related to braids and 3-manifolds. These notions are generalized into higher dimensions. Surface-knots or surface-links are closed surfaces (two-dimensional manifolds) in Euclidean 4-space, which are related to two-dimensional braids and 4-manifolds. Surface-knot theory treats not only closed surfaces but also surfaces with boundaries in 4-manifolds. For example, knot concordance and knot cobordism, which are also important objects in knot theory, are surfaces in the product space of the 3-sphere and the interval. Included in this book are basics of surface-knots and the related topics of classical knots, the motion picture method, surface diagrams, handle surgeries, ribbon surface-knots, spinning construction, knot concordance and 4-genus, quandles and their homology theory, and two-dimensional braids.

Knots are familiar objects. We use them to moor our boats, to wrap our packages, to tie our shoes. Yet the mathematical theory of knots quickly leads to deep results in topology and geometry. The Knot Book is an introduction to this rich theory, starting from our familiar understanding of knots and a bit of college algebra and finishing with exciting topics of current research. The Knot Book is also about the excitement of doing mathematics. Colin Adams engages the reader with fascinating examples, superb figures, and thought-provoking ideas. He also presents the remarkable applications of knot theory to modern chemistry, biology, and physics. This is a compelling book that will comfortably escort you into the marvelous world of knot theory. Whether you are a mathematics student, someone working in a related field, or an amateur mathematician, you will find much of interest in The Knot Book.

More recently, Khovanov introduced link homology as a generalization of the Jones polynomial to homology of chain complexes and Ozsvath and Szabo developed Heegaard-Floer homology, that lifts the Alexander polynomial. These two significantly different theories are closely related and the dependencies are the object of intensive study. These ideas mark the beginning of a new era in knot theory that includes relationships with four-dimensional problems and the creation of new forms of algebraic topology relevant to knot theory. The theory of skein modules is an older development also having its roots in Jones discovery. Another significant and related development is the theory of virtual knots originated independently by Kauffman and by Goussarov Polyak and Viro in the '90s. All these topics and their relationships are the subject of the survey papers in this book.

From prehistory to the present, knots have been used for purposes both artistic

and practical. The modern science of Knot Theory has ramifications for biochemistry and mathematical physics and is a rich source of research projects for undergraduate and graduate students and professionals alike. Quandles are essentially knots translated into algebra. This book provides an accessible introduction to quandle theory for readers with a background in linear algebra. Important concepts from topology and abstract algebra motivated by quandle theory are introduced along the way. With elementary self-contained treatments of topics such as group theory, cohomology, knotted surfaces and more, this book is perfect for a transition course, an upper-division mathematics elective, preparation for research in knot theory, and any reader interested in knots. Completely up-to-date, illustrated throughout, and written in an accessible style, Knots and Surfaces is an account of the mathematical theory of knots and its interaction with related fields. This is an area of intense research activity, and this text provides the advanced undergraduate with a superb introduction to this exciting field. Beginning with a simple diagrammatic approach, the book proceeds through recent advances to areas of current research. Topics including topological spaces, surfaces, the fundamental group, graphs, free groups, and group presentations combine to form a coherent and highly developed theory with which to explore and explain the accessible and intuitive problems of knots and surfaces. - ;The main theme of this book is the mathematical theory of knots and its interaction with the theory of surfaces and of group presentations. Beginning with a simple diagrammatic approach to the study of knots, reflecting the artistic and geometric appeal of interlaced forms, Knots and Surfaces takes the reader through recent advances in our understanding to areas of current research. Topics included are straightforward introductions to topological spaces, surfaces, the fundamental group, graphs, free groups, and group presentations. These topics combine into a coherent and highly developed theory to explore and explain the accessible and intuitive problems of knots and surfaces. Both as an introduction to several areas of prime importance to the development of pure mathematics today, and as an account of pure mathematics in action in an unusual context, this book presents novel challenges to students and other interested readers. -

There is a sympathy of ideas among the fields of knot theory, infinite discrete group theory, and the topology of 3-manifolds. This book contains fifteen papers in which new results are proved in all three of these fields. These papers are dedicated to the memory of Ralph H. Fox, one of the world's leading topologists, by colleagues, former students, and friends. In knot theory, papers have been contributed by Goldsmith, Levine, Lomonaco, Perko, Trotter, and Whitten. Of these several are devoted to the study of branched covering spaces over knots and links, while others utilize the braid groups of Artin. Cossey and Smythe, Stallings, and Strasser address themselves to group theory. In his contribution Stallings describes the calculation of the groups  $\ln/\ln+1$  where  $l$  is the augmentation ideal in a group ring  $RG$ . As a consequence, one has for each  $k$  an

example of a  $k$ -generator  $l$ -relator group with no free homomorphisms. In the third part, papers by Birman, Cappell, Milnor, Montesinos, Papakyriakopoulos, and Shalen comprise the treatment of 3-manifolds. Milnor gives, besides important new results, an exposition of certain aspects of our current knowledge regarding the 3-dimensional Brieskorn manifolds.

[Selected Lectures Presented at the Advanced School and Conference on Knot Theory and Its Applications to Physics and Biology, ICTP, Trieste, Italy, 11 - 29 May 2009](#)

[Topology of Surfaces, Knots, and Manifolds](#)

[Mathematics with a Twist](#)

[Papers Dedicated to the Memory of R.H. Fox. \(AM-84\)](#)

[Virtual Knots](#)

[Knots, Groups and 3-Manifolds \(AM-84\), Volume 84](#)

[Knots and Links](#)

[An Introduction](#)

[Encyclopedia of Knot Theory](#)

[Formal Knot Theory](#)

[Grid Homology for Knots and Links](#)

This is a foundation for arithmetic topology - a new branch of mathematics which is focused upon the analogy between knot theory and number theory. Starting with an informative introduction to its origins, namely Gauss, this text provides a background on knots, three manifolds and number fields. Common aspects of both knot theory and number theory, for instance knots in three manifolds versus primes in a number field, are compared throughout the book. These comparisons begin at an elementary level, slowly building up to advanced theories in later chapters. Definitions are carefully formulated and proofs are largely self-contained. When necessary, background information is provided and theory is accompanied with a number of useful examples and illustrations, making this a useful text for both undergraduates and graduates in the field of knot theory, number theory and geometry. ?

A selection of topics which graduate students have found to be a successful introduction to the field, employing three distinct techniques: geometric topology manoeuvres, combinatorics, and algebraic topology. Each topic is developed until significant results are achieved and each chapter ends with exercises and brief accounts of the latest research. What may reasonably be referred to as knot theory has expanded enormously over the last decade and, while the author describes important discoveries throughout the

twentieth century, the latest discoveries such as quantum invariants of 3-manifolds as well as generalisations and applications of the Jones polynomial are also included, presented in an easily intelligible style. Readers are assumed to have knowledge of the basic ideas of the fundamental group and simple homology theory, although explanations throughout the text are numerous and well-done. Written by an internationally known expert in the field, this will appeal to graduate students, mathematicians and physicists with a mathematical background wishing to gain new insights in this area.

This invaluable book is an introduction to knot and link invariants as generalised amplitudes for a quasi-physical process. The demands of knot theory, coupled with a quantum-statistical framework, create a context that naturally and powerfully includes a extraordinary range of interrelated topics in topology and mathematical physics. The author takes a primarily combinatorial stance toward knot theory and its relations with these subjects. This stance has the advantage of providing direct access to the algebra and to the combinatorial topology, as well as physical ideas. The book is divided into two parts: Part I is a systematic course on knots and physics starting from the ground up, and Part II is a set of lectures on various topics related to Part I. Part II includes topics such as frictional properties of knots, relations with combinatorics, and knots in dynamical systems. In this third edition, a paper by the author entitled "Functional Integration and Vassiliev invariants" has been added. This paper shows how the Kontsevich integral approach to the Vassiliev invariants is directly related to the perturbative expansion of Witten's functional integral. While the book supplies the background, this paper can be read independently as an introduction to quantum field theory and knot invariants and their relation to quantum gravity. As in the second edition, there is a selection of papers by the author at the end of the book. Numerous clarifying remarks have been added to the text. This exploration of combinatorics and knot theory is geared toward advanced undergraduates and graduate students. The author, Louis H. Kauffman, is a professor in the Department of Mathematics, Statistics, and Computer Science at the University of Illinois at Chicago. Kauffman draws upon his work as a topologist to illustrate the relationships between

knot theory and statistical mechanics, quantum theory, and algebra, as well as the role of knot theory in combinatorics. Featured topics include state, trails, and the clock theorem; state polynomials and the duality conjecture; knots and links; axiomatic link calculations; spanning surfaces; the genus of alternative links; and ribbon knots and the Arf invariant. Key concepts are related in easy-to-remember terms, and numerous helpful diagrams appear throughout the text. The author has provided a new supplement, entitled "Remarks on Formal Knot Theory," as well as his article, "New Invariants in the Theory of Knots," first published in The American Mathematical Monthly, March 1988.

Knot theory is the study of embeddings of circles in space. Peter Cromwell has written a textbook on knot theory designed for use in advanced undergraduate or beginning graduate-level courses. The exposition is detailed and careful yet engaging and full of motivation. Numerous examples and exercises serve to help students through the material, while an instructor's manual is available online. A richly illustrated 2004 textbook on knot theory; minimal prerequisites but modern in style and content.

Bringing together many results previously scattered throughout the research literature into a single framework, this work concentrates on the application of the author's algebraic theory of surgery to provide a unified treatment of the invariants of codimension 2 embeddings, generalizing the Alexander polynomials and Seifert forms of classical knot theory.

[Virtual and Classical](#)

[Knots, Links, Spatial Graphs, and Algebraic Invariants](#)

[Knots, Links, Braids, and 3-manifolds](#)

[Physical and Numerical Models in Knot Theory](#)

[Knots and Physics](#)

[High-dimensional Knot Theory](#)

[Knots, Low-Dimensional Topology and Applications](#)

[The Knot Book](#)

[Applications of Knot Theory](#)

[Knots and Links in Three-Dimensional Flows](#)

LinKnot - Knot Theory by Computer provides a unique view of selected topics in knot theory suitable for students, research mathematicians, and readers with backgrounds in other exact sciences, including chemistry, molecular biology and physics. The book covers basic



notions in knot theory, as well as new methods for handling open problems such as unknotting number, braid family representatives, invertibility, amphicheirality, undetectability, non-algebraic tangles, polyhedral links, and (2,2)-moves. Conjectures discussed in the book are explained at length. The beauty, universality and diversity of knot theory is illuminated through various non-standard applications: mirror curves, fullerenes, self-referential systems, and KL automata.

The Only Undergraduate Textbook to Teach Both Classical and Virtual Knot Theory An Invitation to Knot Theory: Virtual and Classical gives advanced undergraduate students a gentle introduction to the field of virtual knot theory and mathematical research. It provides the foundation for students to research knot theory and read journal articles on their own. Each chapter includes numerous examples, problems, projects, and suggested readings from research papers. The proofs are written as simply as possible using combinatorial approaches, equivalence classes, and linear algebra. The text begins with an introduction to virtual knots and counted invariants. It then covers the normalized f-polynomial (Jones polynomial) and other skein invariants before discussing algebraic invariants, such as the quandle and biquandle. The book concludes with two applications of virtual knots: textiles and quantum computation.

This volume contains the proceedings of the AMS Special Session on Algebraic and Combinatorial Structures in Knot Theory and the AMS Special Session on Spatial Graphs, both held from October 24 – 25, 2015, at California State University, Fullerton, CA. Included in this volume are articles that draw on techniques from geometry and algebra to address topological problems about knot theory and spatial graph theory, and their combinatorial generalizations to equivalence classes of diagrams that are preserved under a set of Reidemeister-type moves. The interconnections of these areas and their connections within the broader field of topology are illustrated by articles about knots and links in spatial graphs and symmetries of spatial graphs in and other 3-manifolds.

Gauss diagram invariants are isotopy invariants of oriented knots in manifolds which are the product of a (not necessarily orientable) surface with an oriented line. The invariants are defined in a combinatorial way using knot diagrams, and they take values in free abelian groups generated by the first homology group of the surface or by the set of free homotopy classes of loops in the surface. There are three main results: 1. The construction of invariants of finite type for arbitrary knots in non orientable 3-manifolds. These invariants can distinguish homotopic knots with homeomorphic complements. 2. Specific invariants of degree 3 for knots in the solid torus. These invariants cannot be generalized for knots in handlebodies of higher genus, in contrast to invariants coming from the theory of skein modules. 2 3. We introduce a special class of knots called global knots, in  $F \times \mathbb{R}$  and we construct new isotopy invariants, called T-invariants, for global knots. Some T-invariants (but not all !) are of finite type but they cannot be extracted from the generalized Kontsevich integral, which is consequently not the universal invariant of finite type for the restricted class of global knots. We prove that T-invariants separate all global knots of a certain type. 3 As a corollary we prove that certain links in  $S^3$  are not invertible without making any use of the link group! Introduction and announcement This work is an introduction into the world of Gauss diagram invariants.

In this paper, I will introduce the mathematical ideas of knots and links, define a function on each that we call the energy of a knot or link, and provide several lemmas and theorems that relate the knot and link energies to basic concepts in knot theory. The main results of this paper will be the proofs of two theorems on the existence of energy minimizers for both knots and links. Finally, I will compare the depth and structure of these two major proofs in order to better understand the nature of the results.

The book is the first systematic research completely devoted to a comprehensive study of virtual knots and classical knots as its integral part. The book is self-contained and contains up-to-date exposition of the key aspects of virtual (and classical) knot theory. Virtual knots were discovered by Louis Kauffman in 1996. When virtual knot theory arose, it became clear that classical knot theory was a small integral part of a larger theory, and studying properties of virtual knots helped one understand better some aspects of classical knot theory and encouraged the study of further problems. Virtual knot theory finds its applications in classical knot theory. Virtual knot theory occupies an intermediate position between the theory of knots in arbitrary three-manifold and classical knot theory. In this book we present the latest achievements in virtual knot theory including Khovanov homology theory and parity theory due to V O Manturov and graph-link theory due to both authors. By means of parity, one can construct functorial mappings from knots to knots, filtrations on the space of knots, refine many invariants and prove minimality of many series of knot diagrams. Graph-links can be treated as "diagramless knot theory": such "links" have crossings, but they do not have arcs connecting these crossings. It turns out, however, that to graph-links one can extend many methods of classical and virtual knot theories, in particular, the Khovanov homology and the parity theory.

This book is an introduction to the remarkable work of Vaughan Jones and Victor Vassiliev on knot and link invariants and its recent modifications and generalizations, including a mathematical treatment of Jones-Witten invariants. It emphasizes the geometric aspects of the theory and treats topics such as braids, homeomorphisms of surfaces, surgery of 3-manifolds (Kirby calculus), and branched coverings. This attractive geometric material, interesting in itself yet not previously gathered in book form, constitutes the basis of the last two chapters, where the Jones-Witten invariants are constructed via the rigorous skein algebra approach (mainly due to the Saint Petersburg school). Unlike several recent monographs, where all of these invariants are introduced by using the sophisticated abstract algebra of quantum groups and representation theory, the mathematical prerequisites are minimal in this book. Numerous figures and problems make it suitable as a course text and for self-study.

[Introductory Lectures on Knot Theory](#)

[Knot Theory and Its Applications](#)

[An Elementary Introduction to the Mathematical Theory of Knots](#)

[An Introduction to the New Invariants in Low-dimensional Topology](#)

[Ideal Knots](#)

[Knots](#)

[Categories of Tangles, Coherence, Categorical Deformations, and Topological Invariants](#)

[LinKnot](#)

[The Existence of Energy Minimizers for Knots and Links](#)

## [Knots and Primes](#)

[American Mathematical Society, Short Course, January 4-5, 2008, San Diego, California](#)

*Knot theory is a classical area of low-dimensional topology, directly connected with the theory of three-manifolds and smooth four-manifold topology. In recent years, the subject has undergone transformative changes thanks to its connections with a number of other mathematical disciplines, including gauge theory; representation theory and categorification; contact geometry; and the theory of pseudo-holomorphic curves. Starting from the combinatorial point of view on knots using their grid diagrams, this book serves as an introduction to knot theory, specifically as it relates to some of the above developments. After a brief overview of the background material in the subject, the book gives a self-contained treatment of knot Floer homology from the point of view of grid diagrams. Applications include computations of the unknotting number and slice genus of torus knots (asked first in the 1960s and settled in the 1990s), and tools to study variants of knot theory in the presence of a contact structure. Additional topics are presented to prepare readers for further study in holomorphic methods in low-dimensional topology, especially Heegaard Floer homology. The book could serve as a textbook for an advanced undergraduate or part of a graduate course in knot theory. Standard background material is sketched in the text and the appendices.*

*This book introduces the study of knots, providing insights into recent applications in DNA research and graph theory. It sets forth fundamental facts such as knot diagrams, braid representations, Seifert surfaces, tangles, and Alexander polynomials. It also covers more recent developments and special topics, such as chord diagrams and covering spaces. The author avoids advanced mathematical terminology and intricate techniques in algebraic topology and group theory. Numerous diagrams and exercises help readers understand and apply the theory. Each chapter includes a supplement with interesting historical and mathematical comments.*

*This proceedings volume presents a diverse collection of high-quality, state-of-the-art research and survey articles written by top experts in low-dimensional topology and its applications. The focal topics include the wide range of historical and contemporary invariants of knots and links and related topics such as three- and four-dimensional manifolds, braids, virtual knot theory, quantum invariants, braids, skein modules and knot algebras, link homology, quandles and their homology; hyperbolic knots and geometric structures of three-dimensional manifolds; the mechanism of topological surgery in physical processes, knots in Nature in the sense of physical knots with applications to polymers, DNA enzyme mechanisms, and protein structure and function. The contents is based on contributions presented at the International Conference on Knots, Low-Dimensional Topology and Applications - Knots in Hellas 2016, which was held at the International Olympic Academy in Greece in July 2016. The goal of the international conference was to promote the exchange of methods and ideas across disciplines and generations, from graduate students*

to senior researchers, and to explore fundamental research problems in the broad fields of knot theory and low-dimensional topology. This book will benefit all researchers who wish to take their research in new directions, to learn about new tools and methods, and to discover relevant and recent literature for future study.

This book, written by a mathematician known for his own work on knot theory, is a clear, concise, and engaging introduction to this complicated subject, and a guide to the basic ideas and applications of knot theory. 63 illustrations.

This book provides an introduction to hyperbolic geometry in dimension three, with motivation and applications arising from knot theory. Hyperbolic geometry was first used as a tool to study knots by Riley and then Thurston in the 1970s. By the 1980s, combining work of Mostow and Prasad with Gordon and Luecke, it was known that a hyperbolic structure on a knot complement in the 3-sphere gives a complete knot invariant. However, it remains a difficult problem to relate the hyperbolic geometry of a knot to other invariants arising from knot theory. In particular, it is difficult to determine hyperbolic geometric information from a knot diagram, which is classically used to describe a knot. This textbook provides background on these problems, and tools to determine hyperbolic information on knots. It also includes results and state-of-the art techniques on hyperbolic geometry and knot theory to date. The book was written to be interactive, with many examples and exercises. Some important results are left to guided exercises. The level is appropriate for graduate students with a basic background in algebraic topology, particularly fundamental groups and covering spaces. Some experience with some differential topology and Riemannian geometry will also be helpful. Almost since the advent of skein-theoretic invariants of knots and links (the Jones, HOMFLY, and Kauffman polynomials), the important role of categories of tangles in the connection between low-dimensional topology and quantum-group theory has been recognized. The rich categorical structures naturally arising from the considerations of cobordisms have suggested functorial views of topological field theory. This book begins with a detailed exposition of the key ideas in the discovery of monoidal categories of tangles as central objects of study in low-dimensional topology. The focus then turns to the deformation theory of monoidal categories and the related deformation theory of monoidal functors, which is a proper generalization of Gerstenhaber's deformation theory of associative algebras. These serve as the building blocks for a deformation theory of braided monoidal categories which gives rise to sequences of Vassiliev invariants of framed links, and clarify their interrelations.

Contents: Knots and Categories: Monoidal Categories, Functors and Natural Transformations; A Digression on Algebras; Knot Polynomials; Smooth Tangles and PL Tangles; A Little Enriched Category Theory; Deformations: Deformation Complexes of Semigroupal Categories and Functors; First Order Deformations; Units; Extrinsic Deformations of Monoidal Categories; Categorical Deformations as Proper Generalizations of Classical Notions; and other papers. Readership:

*Mathematicians and theoretical physicists.*

*Knot theory is a kind of geometry, and one whose appeal is very direct because the objects studied are perceivable and tangible in everyday physical space. It is a meeting ground of such diverse branches of mathematics as group theory, matrix theory, number theory, algebraic geometry, and differential geometry, to name some of the more prominent ones. It had its origins in the mathematical theory of electricity and in primitive atomic physics, and there are hints today of new applications in certain branches of chemistry. The outlines of the modern topological theory were worked out by Dehn, Alexander, Reidemeister, and Seifert almost thirty years ago. As a subfield of topology, knot theory forms the core of a wide range of problems dealing with the position of one manifold imbedded within another. This book, which is an elaboration of a series of lectures given by Fox at Haverford College while a Philips Visitor there in the spring of 1956, is an attempt to make the subject accessible to everyone. Primarily it is a text book for a course at the junior-senior level, but we believe that it can be used with profit also by graduate students. Because the algebra required is not the familiar commutative algebra, a disproportionate amount of the book is given over to necessary algebraic preliminaries.*

[Quandles](#)

[Knots and Surfaces](#)

[The State of the Art](#)

[Surface-Knots in 4-Space](#)

[An Invitation to Knot Theory](#)

[An Introduction to Knot Theory](#)

[Introduction to Knot Theory](#)